WEEK 1

III

REMEMBER:

Number = Base^{Power}

The laws of indices

In the following laws a is the common *base*, m and n are the *indices* (exponents). Each law has an example of its use alongside.

1. $a^m \times a^n = a^{m+n}$ $\quad 2^2 \times 2^4 = 2^{2+4} = 2^6 = 64$
2. $\frac{a^m}{a^n} = a^{m-n}$ $\quad \frac{3^4}{3^2} = 3^{4-2} = 3^2 = 9$
3. $(a^m)^n = a^{mn}$ $\quad (2^2)^3 = 2^{2\times3} = 2^6 = 64$
4. $a^0 = 1$ $\quad$ Any number raised to the power 0 is always 1
5. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $\quad 27^{\frac{4}{3}} = \sqrt[3]{27^4} = 3^4 = 81$
6. $a^{-n} = \frac{1}{a^n}$ $\quad 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

We need to study these laws carefully in order to understand the significance of each.

Law 1 you have already meet, it enables us to *multiply numbers* given in index form that have a common base. In the example the common base is 2, the first number raises this base (factor) to the power 2 and the second raises the same base to the power 3. In order to find the result we simply *add* the indices.

Law 2 we have again used when *dividing numbers* with a common base, in this case the base is 3. Note that since division is the opposite arithmetic operation to multiplication, it follows that we should perform the opposite arithmetic operation on the indices, that of *subtraction*. Remember we always subtract the index in the denominator from the index in the numerator.

Law 3 is concerned with raising the powers of numbers. Do not mix this law up with law 1. When *raising powers of numbers* in index form, we *multiply* the indices.

Law 4 you have also met; this law simply states that *any number raised to the power 0 is always 1*. Knowing that any number divided by itself is also 1, we can use this fact to show that a number raised to the power 0 is also 1. What we need to do is use the second law concerning the division of numbers in index form.

We know that $\frac{9}{9} = 1$ or $\frac{3^2}{3^2} = 3^{2-2} = 3^0 = 1$ which shows that $3^0 = 1$ and in fact because we have used the second law of indices, this must be true in all cases.

Key point

When you move a number in index form above or below the fraction line always change the sign of the index.

Law 5 is a rather complicated looking law, which enables us to find the decimal equivalent of a number in index form, where the index is a fraction. All that you need to remember is that the index number above the fraction line is raised to that power and the index number below the fraction line has that number root.

So for the number, $8^{\frac{2}{3}}$ then we raise 8 to the power 2 and then take the cube root of the result. It does not matter in which order we perform these operations. So we could have just as easily taken the cube root of 8 and then raised it to the power 2.

Law 6 is very useful when you wish to convert the division of a number to multiplication. In other words bring a number from underneath the division line to the top of the division line. *As the number crosses the line we change the sign of its index.* This is illustrated in the example which accompanies this law.

The following examples, further illustrate the use of the above laws, when evaluating or simplifying expressions that involve numbers and symbols.

IV

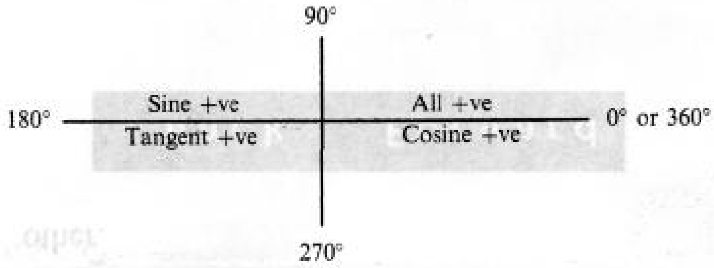
Trigonometric fundamentals

Formulae

(1) Radian measure

(i) π radians $= 180°$,
(ii) if θ is small and measured in radians, then $\sin\theta = \theta$, $\cos\theta = 1$, $\tan\theta = \theta$,
(iii) arc length $s = r\theta$ (θ in radians),
(iv) area of sector $= \frac{1}{2}r^2\theta$ (θ in radians).

(2) Trigonometric ratios for angles of any magnitude



For all values of θ: $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(3) Equivalent ratios

	$90° \le \theta \le 180°$	$180° \le \theta \le 270°$	$270° \le \theta \le 360°$
$\sin\theta =$	$\sin(180° - \theta)$	$-\sin(\theta - 180°)$	$-\sin(360° - \theta)$
$\cos\theta =$	$-\cos(180° - \theta)$	$-\cos(\theta - 180°)$	$\cos(360° - \theta)$
$\tan\theta =$	$-\tan(180° - \theta)$	$\tan(\theta - 180°)$	$-\tan(360° - \theta)$

(4) Triangle formulae

(i) Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2r$

(ii) Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

(iii) Area $= \frac{1}{2}bc\sin A$

$= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{(a+b+c)}{2}$

(5) Polar and Cartesian co-ordinate system

(i) $x = r\cos\theta$ $y = r\sin\theta$

(ii) $r = +\sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left[\dfrac{y}{x}\right]$, $\tan\theta = \dfrac{y}{x}$

(6) Equation of a circle, centre at the origin, radius (a)

$$x^2 + y^2 = a^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where the centre is at $(-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$

(7) Superposition

(i) $a\sin\omega x + b\cos\omega x = c\sin(\omega x + \phi)$ where point (a, b) has polar co-ordinates r, θ and $c = r = \sqrt{a^2 + b^2}$ and $\tan\phi = \dfrac{b}{a}$

WEEK 1

Maths
Indices Questions.

1. factors of 16
2. Simplify $\frac{1}{2^3} \times 2^7 \times \frac{1}{2^{-5}} \times 2^{-4}$
3. Simplify $\left(\frac{16}{81}\right)^{\frac{3}{4}}$
4. Rewrite $47.7 \mu f$
   $70 GPa$
5. Evaluate $2\frac{5}{8} + \frac{7}{16} - \frac{3}{8}$

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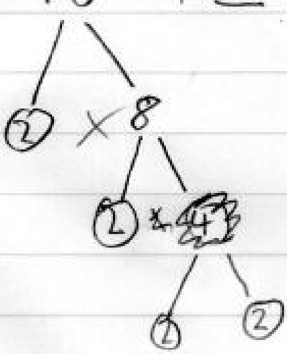
1. $16 = 2^4$



16×1
2×8
4×4

2. $\cancel{\frac{1}{2^3}} \times 2^7 \times \cancel{\frac{1}{2^{-5}}} \times 2^{-4}$

$2^{-3} \times 2^{7} \times 2^{5} \times 2^{-4} = 2^5$

3. $\left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$ — Multiply out.

$\frac{2^{4 \times \frac{3}{4}}}{3^{4 \times \frac{3}{4}}} = \frac{2^3}{3^3} = \frac{8}{27}$

4. $47.7 \mu f = 47.7 \times 10^{-6} f$

$70 GPa = 70 \times 10^9 Pa$

5. $2\frac{5}{8} + \frac{7}{16} - \frac{3}{8} = \frac{21}{8} + \frac{7}{16} + \frac{3}{8} = \frac{42 + 7 - 6}{16}$

WEEK 2

Indices Revision

$$\frac{x^3 \times x^4}{\sqrt{x^8}} = \frac{x^7}{\sqrt{x^8}} = \frac{x^7}{(x^8)^{\frac{1}{2}}}$$

$x^0 = 1$

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^1 = x$

$\sqrt{x} \times \sqrt{x} = x$

$$= \frac{x^3 \times x^4}{(x^8)^{\frac{1}{2}}}$$

$$= x^3 \times x^4 \times (x^8)^{-\frac{1}{2}}$$

$$= x^3 \times \cancel{x^4} \times \cancel{x^{-4}}$$

$$= \underline{x^3}$$

① $z^4 \times z^2 \times z^{-3}$

$= z^6 \times z^{-3}$

$= \underline{z^3}$ ✓

② $n^8 \div n^5 = \frac{n^8}{n^5} = \underline{n^3}$ ✓

③ $10^5 \times 10^3 \div 10^4$

$= 10^8 \div 10^4$

$= \underline{10^4}$ ✓

④ $\left(\frac{a^2}{a^5}\right)^3 = a^6 \div a^{15}$ $\div$!

$= \underline{a^{-9}}$

$= (a^{-3})^3$

$= \underline{a^{-9}}$ ✓

⑤ $4 \div 4^{\frac{1}{2}} = 4^1 \times 4^{-\frac{1}{2}} = \underline{4^{\frac{1}{2}}}$ ✓

$x^1 \div x^{-\frac{1}{2}}$

(1)

Cambridge Problem

Q) Solve $\sqrt{3-2\sqrt{2}} = ?$ options ① $\sqrt{3}-1$, ② $\sqrt{2}-1$ or ③ $\sqrt{3}-\sqrt{2}$

A) So $\sqrt{3-2\sqrt{2}}$

perfect square rule :-

try: $A^2 - 2AB + B^2 = (A-B)^2$ (factors)

what are A & B?

$3-2\sqrt{2} = (\sqrt{2})^2 - 2\sqrt{2}\cdot 1 + 1^2 = (\sqrt{2}-1)^2$

$2+1=3$ $B=1$

Original form $\sqrt{3-2\sqrt{2}} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2}-1$

squares cancel

option ②

ANSWER is ②.