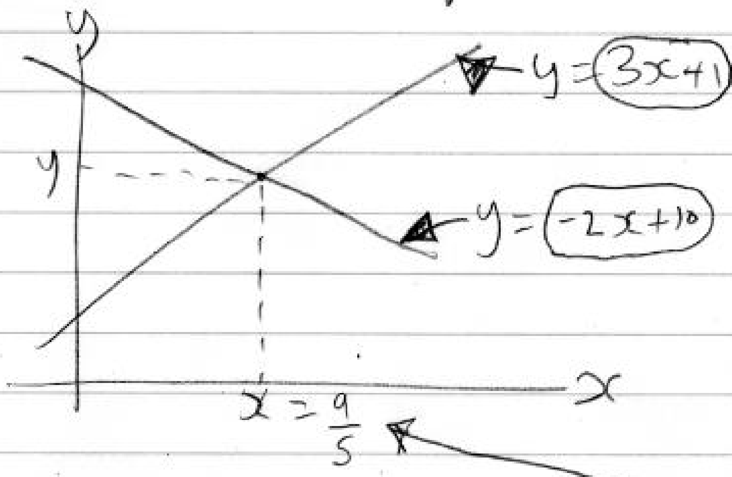
Simultaneous equations and Quadratic equations

WK10
20/11

Simultaneous Equations.



$y = 3x + 1$

$y = -2x + 10$

$0 = 3x + 2x + 1 - 10$

$0 = 5x - 9$

$x = \frac{9}{5}$

Revision: $E = \frac{1}{2} M V^2$ Solve for V;

$2 \times E = \cancel{\frac{1}{2}} M V^2 \cancel{\times 2}$

(÷m) $\frac{2E}{M} = \frac{\cancel{M} V^2}{\cancel{M}}$ (÷m)

$\sqrt{\frac{2E}{M}} = \sqrt{V^2}$ (√ square both sides)

$\sqrt{\frac{2E}{M}} = V$

Remember!

$2^4 = 4$

$\therefore \sqrt[2]{4} = 2$

example 2:

$2^3 = 8$

$\therefore \sqrt[3]{8} = 2$

Another example;

$A = \pi R^2$ for R

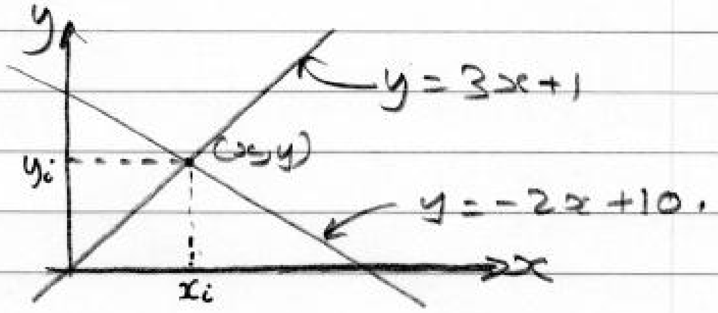
(÷π) $\frac{A}{\pi} = \frac{\cancel{\pi} R^2}{\cancel{\pi}}$ (÷π)

$\sqrt{\frac{A}{\pi}} = \sqrt{R^2}$ (√)

$\therefore R = \sqrt{\frac{A}{\pi}}$

Simultaneous equations

$y = mx + c$



Solving pairs of simultaneous equations:

$(3x+1)+(2x+10) = 0$

$\therefore\ 3x+1 = -2x+10$

$\quad +2x \qquad +2x$

$5x + 1 = 10$

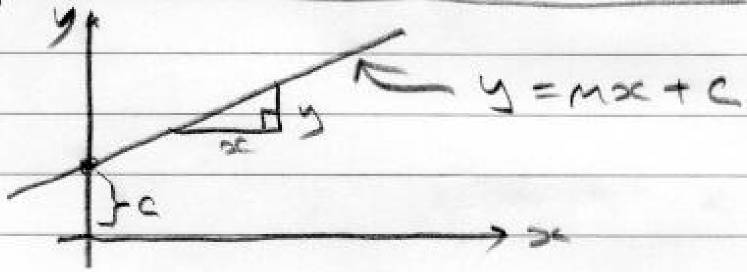
$\quad -1 \qquad -1$

$5x = +9 \longrightarrow x_i = \frac{9}{5}$

$(\div 5) \qquad (\div 5)$

$\therefore\ x_i = 1.8$

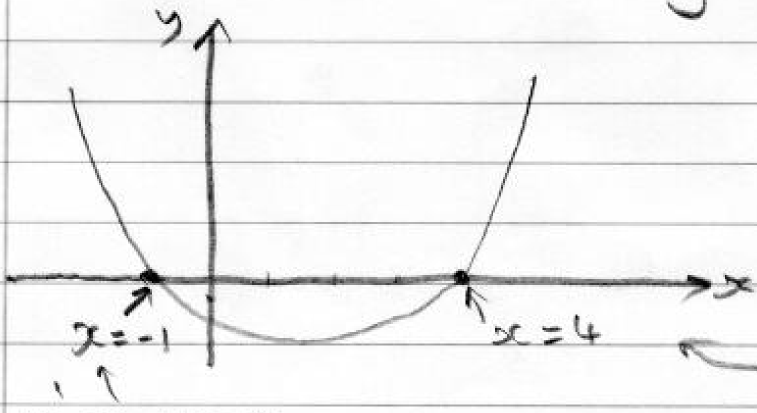
Graphical Representation formulae:



$\therefore\ m = \frac{y}{x}$

Quadratic:

$y = ax^2 + bx + c$



$\therefore\ x - 4 = 0$

$\therefore\ x + 1 = 0$

Standard form of a quadratic equation;

$$ax^2 + bx + c = 0$$

Solution for x;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(x+1)(x-4) = 0$$

$x^2 \quad +1x \quad -4x \quad -4$

$$\therefore \quad x^2 - 3x - 4 = \cancel{0}\ y$$

$(x+1)(x-4)$

$x^2 + 1x$
$-4x - 4$

$x^2 - 3x - 4$

example 2

$$x^2 - 4x + 4 = 0$$

$x^2 \quad 1x \quad -4x + 4$

$(x \cancel{4})(x \cancel{+4})$

$-1 \qquad +5$

$x^2 \quad -1x \quad +5x \quad -4x \quad +4$

$x^2 + 5x$
$\pm \quad -1x \quad +4$

$x^2 - 4x + 4$

example 3

$$x^2 - \frac{1}{9} = 0$$

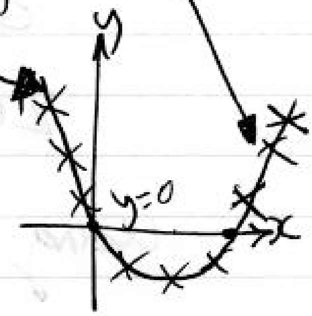
$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$x^2 - \left(\frac{1}{3}\right)^2 = \left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right)$$

Quadratic equations.

$$\left(ax^2 + 6x + c = 0 \atop = y \right)$$

$ax^2 + 6x + c = 0 = y$ parabola.



Q1/ $x^2 - 6x + 9 = y$

$x^2 - 6x + 9 = 0$

factorise: $(x - 3)(x - 3) = 0$

$-3 + -3$ $\quad -3x - 3 = 9$

$x^2 - 6x + 9$ ✓

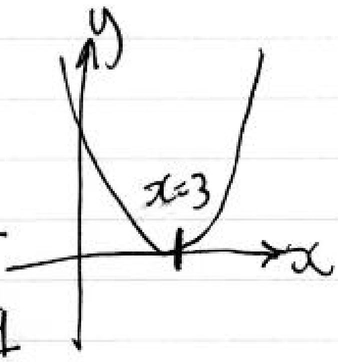
either $(x - 3 = 0)$ $\quad x = 3$

or $(x - 3 = 0)$

$x - 3$
$x - 3$

$x^2 - 3x$
$-3x + 9$

$x^2 - 6x + 9$



Q2/ $x^2 - 16 = 0 \quad = x^2 + 0x - 16 = 0$

factorise: $(x + 4)(x - 4) = 0$

$+4x - 4 = -16$

$x + x = x^2$

$x + 4 = 0 \therefore x = -4$

$x - 4 = 0 \therefore x = 4$

$(x\_)(x\_)$

Q3/ $x^2 - 3x + 2 = 0$

factorise $(x - 1.5)(x - 1.5) = 0$

$x^2 - 1.5$

$-1.5 + 2$

$+ + = +$

$- - = +$

Ex/ $12t^2 + 6t - 60 = 0$

Solve for t.

$$\frac{12t^2}{6} + \frac{6t}{6} - \frac{60}{6} = 0$$

$$2t^2 + t - 10 = 0 \quad \leftarrow ax^2 + bx + c \text{ form}$$

Using quadratic formula; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-10)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{1 + 80}}{4}$$

$$\therefore t = \frac{-1 \pm 9}{4}$$

$$t = \frac{-1+9}{4} \quad \text{or} \quad \frac{-1-9}{4}$$

$$= \underline{\underline{2}} \qquad = \underline{\underline{-2.5}}$$

$$\therefore \underline{\underline{t = 2, -2.5}}$$

EXCEL: FORMULAS

D2.1 A) $\boxed{fx}$ ← formula functions.

values of x	(−x)	exp	
0 (A3)	B3 (=−A3)	EXP (=EXP(B3))	=10*(1−EXP)
0.05			
5			

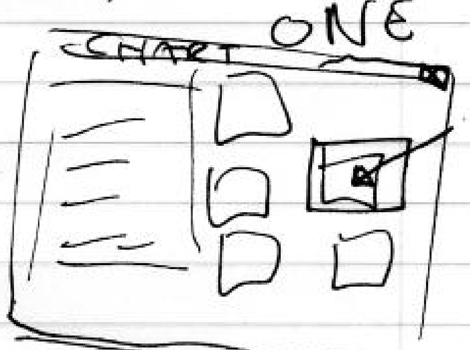
=EXP(B3) function

10*(1−EXP)

① TABLE

② Select All A+D collums.

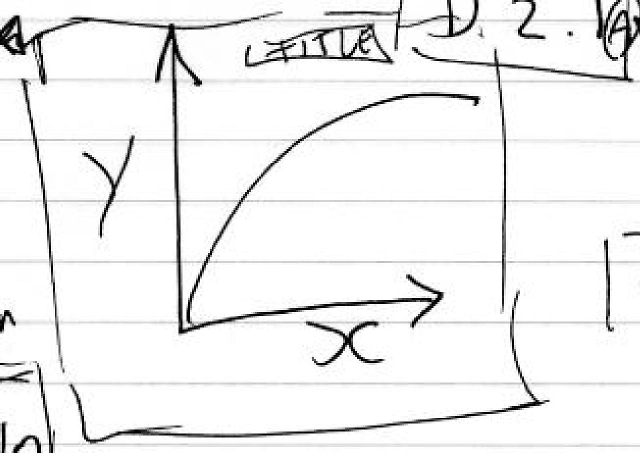
③ CHART FUNCTION XY − SMOOTH ONE



④ OK

⑤ format chart area etc... font/coloured.. → LABLE → Exponential growth (+ decay) graph

⑥ PRINT



D.2.2(b)

D2.2. LN − log base n

LOG BASE10

= LOG(A3,10)

END OF SECTION TEST

Q **2** A train is accelerating along a level track.
The distance, s, travelled by the train as a function of time, t, is given by the equation $s = 7t + 2t^2$.

Calculate the time taken for the train to travel a distance of 15 m.

When answering 'Calculate' questions, you need to find the number or amount of something using the information provided in the question. This might involve applying a particular technique, mathematical method or formula.

Guided

Substitute $s = 15$ into the equation to give:

$15 = 7t + 2t^2$

Rearrange into the general form of a quadratic, making one side equal to zero:

$2t^2 + 7t - 15 = 0$

Rewrite the equation in the form $2t^2 + 10t - 3t - 15 = 0$

Then: $2t(t + \ldots\ldots\ldots) - 3(t + \ldots\ldots\ldots) = 0$

Take $(t + \ldots\ldots\ldots)$ as a common factor to give:

$(t + \ldots\ldots\ldots)(2t - 3) = 0$

Equate each of the brackets to zero and find two possible values of t:

$(2t - \ldots\ldots\ldots) = 0$ so $t = \ldots\ldots\ldots\ldots\ldots$

$(t + \ldots\ldots\ldots) = 0$ so $t = \ldots\ldots\ldots\ldots\ldots$

Check values for t by substituting back into original equation:

....

....

The negative value of t is not a feasible response in the given scenario.

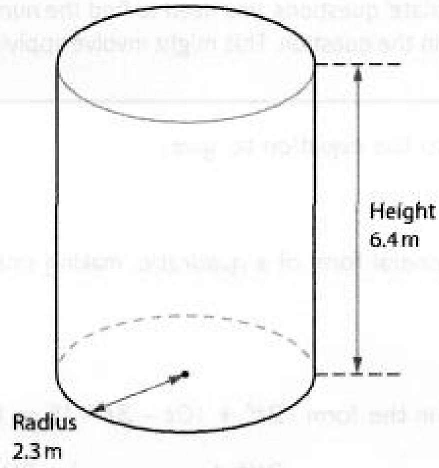
Answer: The time taken for the train to travel 15 m is ....

Links See pages 7–8 of the Revision Guide to revise solving quadratic equations.

Total for Question 2 = 2 marks

AREA'S + VOLUME'S

Q **3** **An engineering company manufactures storage tanks from sheet metal. A new design for an open-topped cylindrical tank needs to be analysed to determine the amount of material required in its construction.**



Calculate the external surface area of the open-topped tank (the material thickness is negligible).

Guided

From the formula booklet, total surface area of a cylinder is: ....

....

But this includes both top and bottom of a fully enclosed cylinder. In this case, only the bottom is required, so the surface area is: ....

....

Substituting $h = 6.4$ and $r = 2.3$ gives: ....

....

....

Answer: The surface area of the open topped tank A = ... m^2

Links See page 14 of the Revision Guide to revise finding the surface area and volume of regular shapes.

Total for Question 3 = 2 marks