Polynomial Division:

NB:

Cubic

$f(x) = x^3 - 7x - 6 = 0$ $\quad \frac{x^3}{x} = x^2$

$(x\quad)(x\quad)(x\quad)$

guess
Let $x = 3$ ∴ $3^3 - 7(3) - 6 = 0$

∴ $\underline{x = -3}$

use polynomial division:-

$$\frac{x^3 - 7x - 6}{x - 3} \Rightarrow (x-3)\overline{)\,x^3 + 0x^2 - 7x - 6}$$

$x^2 + 3x + 2$

$ax^2 + bx + c = 0$

OR

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$(x-3) \times x^2 \rightarrow -x^3 - 3x^2$

$0 + 3x^2 - 7x - 6$

$3x \times x = 3x^2$ $\rightarrow +3x^2 - 9x$

$-3x + 3 = -9$

$0 + 2x - 6$

$+2x - 6$

$0 - 0$

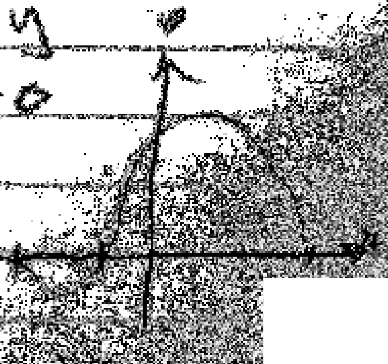
$(x-3)(x^2 + 3x + 2)$

factorise:-

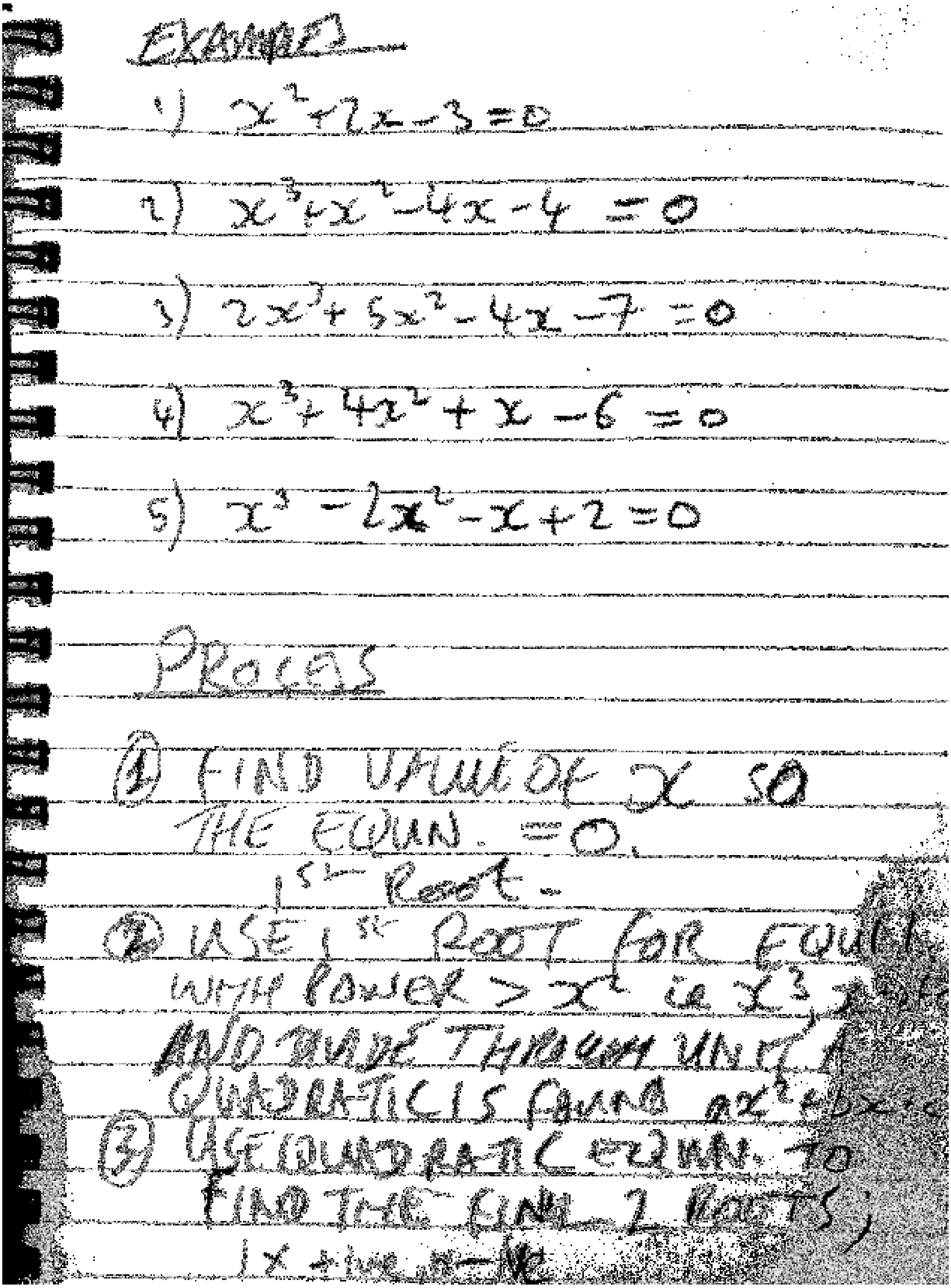
1. $(x-3)(x+2)(x+1) = 0$

$\downarrow \quad \downarrow \quad \downarrow$

then $x = 3$ $\quad x = -2$ $\quad x = -1$



Ans $x^3 - 7x - 6 = (x-3)(x+1)(x+1)$



EXAMPLES

1) $x^2+2x-3=0$

2) $x^3+x^2-4x-4=0$

3) $2x^3+5x^2-4x-7=0$

4) $x^3+4x^2+x-6=0$

5) $x^3-2x^2-x+2=0$

PROCESS

① FIND VALUE OF x SO THE EQUN. $=0$.
1st Root.

② USE 1st ROOT FOR EQUN. WITH POWER $> x^2$ ie x^3,
AND DIVIDE THROUGH UNTIL A QUADRATIC IS FOUND ax^2+bx+c

③ USE QUADRATIC EQUN. TO FIND THE FINAL 2 ROOTS;
1x +ve, x −ve

Q1:

POLYNOMIAL DIVISION

1 $x^2+2x-3=0$ find roots.

QUADRATIC. $\Rightarrow x^2$ form $ax^2+bx+c=0$.

Set $f(x)=2$ $\therefore 2^2+2(2)-3=$

$=4+4-3=5 \neq \underline{0}.$

Set $f(x)=1$ $\therefore 1^2+2(1)-3=$

$=1+2-3=\underline{\underline{0}}.$ $\therefore (x-1)=0$ is a factor.

polynomial division for the other 2 terms;

$$\frac{x^2+2x^1-3}{x^1-1} \rightarrow 1x^1-1\,\overline{)\,1x^2+2x^1-3}$$

$1x^1 \times 1x^1 = 1x^2 \rightarrow$ $1x^2-1x$

$0+3x-3$

$+3x-3$

0

Quotient: $1x^1+3$

NOTE:–

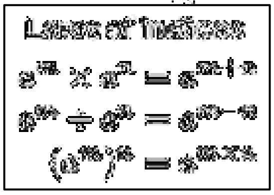
$- \; - = +$

$+ \; + = +$

$- \; + = -$

$+ \; - = -$

Remember Laws of Indices


Laws of Indices

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{m \times n}$

Remainder theorem

-1 X 1x¹ = 1x

$\therefore 1x^1+3 = (x+3)$

Alternative method;

Peter's method;

$$\frac{x^2+2x-3}{x-1} = \frac{1x^2}{1x^1-1} + \cancel{\frac{2x^1}{1x^1-1}} + \frac{-3}{1x^1-1}$$

Divide through by the 1st root found, to simplify the equation from an x^3 to an x^2 eqn, which we know is a quadratic and can be solved to find the remaining 2 roots

$\underbrace{(x^1+0)+ \quad (0)}\; + (0x^1+3)$

$\underline{\underline{x \quad + 3}}$

Second factor is +3;

1x² / 1x¹ = 1(1)x²⁻¹ = x

Factorise:

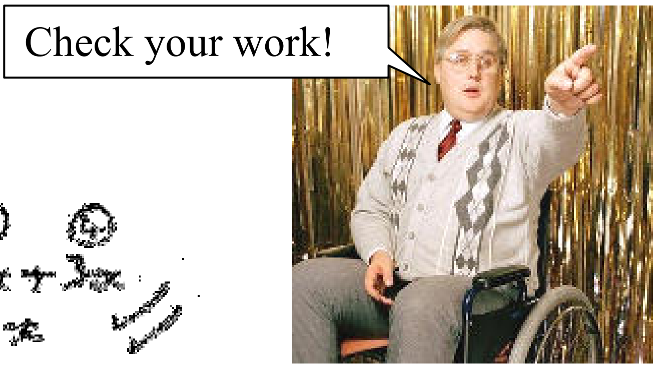
Check:

☺ ☑ ☞ $(x-1)(x+3)$ ① ② ③ ④

① ② ③ ④

$x^2+(1\times 3)-1x+3x$

x^2-3+2x //



Thus;

$$x^2+2x-3 = (x-1)(x+3)$$

ANSWER: $\underline{\underline{\qquad}}$

1) $x^2+2x-3=0$ find roots. QUADRATIC. $\Rightarrow x^2 \uparrow$

form $ax^2+bx+c=0$.

ALT. METHOD USING QUADRATIC FORMULA;

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

where $a=1$
$b=2$
$c=3$

$$=\frac{-2\pm\sqrt{2^2-4(1)(3)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{16}}{2}$$

+ive Root; $=\frac{-2+4}{2}=1$

−ive Root; $=\frac{-2-4}{2}=-3$

ANSWER:

$\therefore x^2+2x-3=\underline{\underline{(x+1)(x-3)}}$.

Q2:

POLYNOMIAL DIVISION

2) $x^3+x^2-4x-4=0$ 3 roots $\therefore x^3 \uparrow$

Set $x=4 \therefore f(4)=(4)^3+4^2-4(4)-4=$
$= 64 \cancel{+16} \cancel{-16} -4 = \underline{60}$

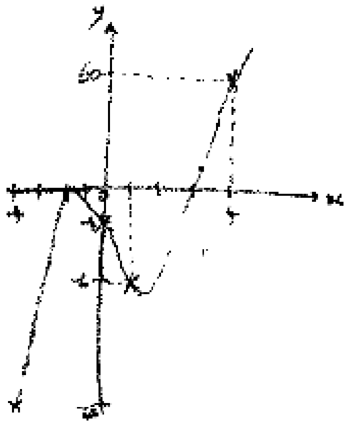
Set $x=1 \therefore f(1)=1^3+1^2-4(1)-4=$
$= 1 + 1 - 4 - 4 = \underline{\underline{-6}}$

Set $x=0 \therefore f(0)=0^3+0^2-4(0)-4=\underline{\underline{-4}}$

Set $x=-4 \therefore f(-4)=(-4)^3+(-4)^2-4(-4)-4=$
$= -64 \cancel{-16} \cancel{+16} -4 = \underline{\underline{-68}}$

Set $x=-2 \therefore f(-2)=(-2)^3+(-2)^2-4(-2)-4=$
$= \cancel{-8} \cancel{-4} \cancel{+8} \cancel{-4}$

$\therefore x=-2,\ f(x)=0$



Set a value of x, i.e. x = -2, when substituted into the x^2 term, cancels out the constant (+c) term, i.e. -4 here, thus will reveal one of the roots of the equation

1

$\therefore$ 1 root; $(x-2)$ is a factor

Use polynomial division to find the other 2 root's;

$$\therefore \frac{x^3+x^2-4x-4}{x-2} = \frac{1x^3}{1x^1-2} + \frac{1x^2}{1x^1-2} - \frac{4x^1}{1x^1-2} - \frac{4}{1x^1-2}$$

$$= 1x^2 + 1x^1 \underbrace{-4 \quad +2}$$

$$= x^2 + x - 2$$ Quadratic eqn:

$ax^2+bx+c=0$

$$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\therefore a=1, \; b=1, \; c=-2$

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(-2)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$+\text{ive}' = \frac{-1+\sqrt{9}}{2} = \frac{-1+3}{2} = 1$$

$$-\text{ive}' = \frac{-1-\sqrt{9}}{2} = \frac{-1-3}{2} = -2$$

Roots: $= (x-1)(x+2)$

☺ ☑ ☞ Check

$(x-1)(x+2)$

$x^2 - 2 \underbrace{- 1x + 2x}$

$x^2 - 2 + x$

$(x-2)(x^2-2+x)$

$x^3 \underbrace{-2x} + \underline{1x^2} - \underline{2x^2} + 4 \underline{-2x}$

$\underline{x^3 - 4x - x^2 + 4}$ ✓✓

ANSWER:

$\therefore x^3 + x^2 - 4x - 4 = (x-2)(x-1)(x+2)$

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Q3:

POLYNOMIAL DIVISION

3) $2x^3+5x^2-4x-7=0$ 3 roots ∴ x^3↑

ie $x=3$; $f(x)=0$

$2(3)^3+5(3)^2-4(3)-7=$

$54+45-12-7=80 \neq 0$ ×

SET $x=0$

$\cancel{2(0)^3}+\cancel{5(0)^2}-\cancel{4(0)}-7=-7 \neq 0.$ ×

Set $x=1$

$2(1)^3+5(1)^2-4(1)-7=$

$2+5-4-7=-4 \neq 0.$ getting closer.

Set $x=2$

$2(2)^3+5(2)^2-4(2)-7=$

$16+20-8-7=21$

Set $x=-1$

$-1\times-1=+1$

$2(-1)^3+5(-1)^2-4(-1)-7$

$-2+5+4-7$

$=0$ CORRECT THUS, in summary;

$f(x)$

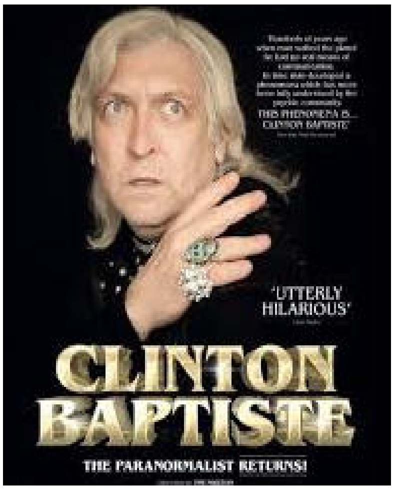
$x=3 \rightarrow 80$

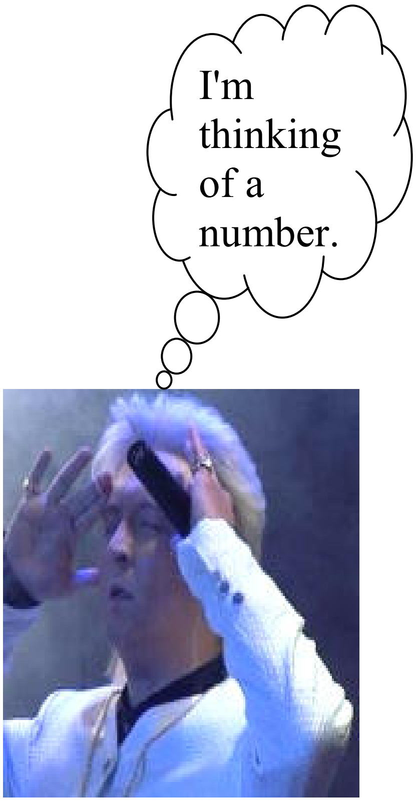
$x=2 \rightarrow 21$

$x=1 \rightarrow -4$

$x=0 \rightarrow -7$

$x=-1 \rightarrow 0$





∴ 1 root ; $(x-1)$ is a factor

Use polynomial division to find the other 2 root's ;

$$\therefore \frac{2x^3+5x^2-4x-7}{(x-1)} =$$

$$\begin{array}{r|l} & 2x^2 + 3x - 7 \\ \hline x-1 & 2x^3 + 5x^2 - 4x - 7 \\ & -\underline{2x^3 \mp 2x^2} \\ & 0 + 3x^2 - 4x \\ & -\underline{3x^2 \mp 3x} \\ & 0 - 7x - 7 \\ & -\underline{-7x \mp 7} \\ & 0 \quad 0 \end{array}$$

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

$$\boxed{a^0 = 1}$$

Zeros

$$= 2x^2 + 3x - 7$$

use;

$$ax^2+bx+c=0$$

$$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$ax^2+bx+c = 2x^2 + 3x - 7$$

$$x = \frac{-3 \pm \sqrt{3^2-4(2)(-7)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9-(-56)}}{4} = \frac{-3 \pm \sqrt{65}}{4}$$

$$= \frac{-3 \pm 8.062}{4}$$

Careful with the signs during the transposition

$$+\text{ive} = \frac{-3 + 8.062}{4} = 1.266$$

$$-\text{ive} = \frac{-3 - 8.062}{4} = -2.766$$

ANSWER:

$$\therefore 2x^3 + 5x^2 - 4x - 7 = (x - - 1)(x + 1.26)(x - 2.76)$$

Others;

4. $2x^3 - x^2 - 16x + 15$

$[(x-1)(x+3)(2x-5)]$

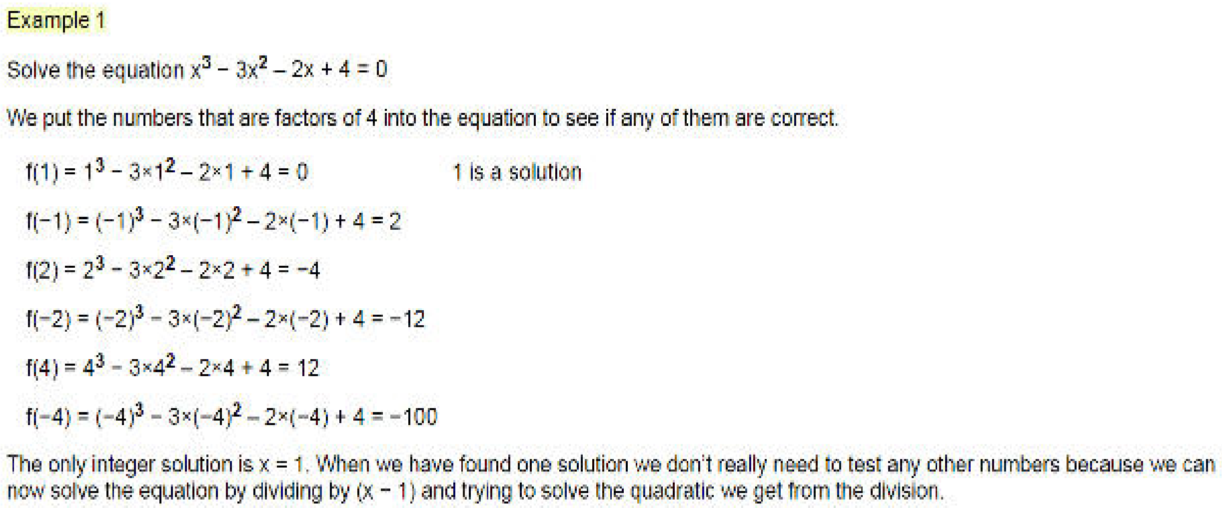
5. Use the factor theorem to factorise $x^3 + 4x^2 + x - 6$ and hence solve the cubic equation $x^3 + 4x^2 + x - 6 = 0$

$$\left[\begin{array}{l} x^3 + 4x^2 + x - 6 \\ \qquad = (x-1)(x+3)(x+2); \\ \qquad x = 1, x = -3 \text{ and } x = -2 \end{array}\right]$$

6. Solve the equation $x^3 - 2x^2 - x + 2 = 0$

$[x = 1, x = 2 \text{ and } x = -1]$

Examples of Cubic and quartic equations can be found at http://www.rasmus.is/uk/t/F/Su52k02.htm



Example 1

Solve the equation $x^3 - 3x^2 - 2x + 4 = 0$

We put the numbers that are factors of 4 into the equation to see if any of them are correct.

$f(1) = 1^3 - 3\times1^2 - 2\times1 + 4 = 0$ 1 is a solution

$f(-1) = (-1)^3 - 3\times(-1)^2 - 2\times(-1) + 4 = 2$

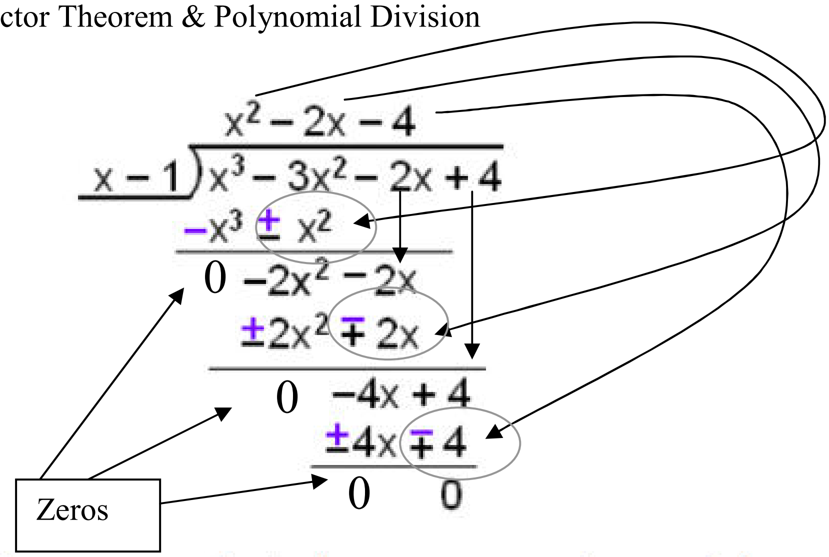
$f(2) = 2^3 - 3\times2^2 - 2\times2 + 4 = -4$

$f(-2) = (-2)^3 - 3\times(-2)^2 - 2\times(-2) + 4 = -12$

$f(4) = 4^3 - 3\times4^2 - 2\times4 + 4 = 12$

$f(-4) = (-4)^3 - 3\times(-4)^2 - 2\times(-4) + 4 = -100$

The only integer solution is x = 1. When we have found one solution we don't really need to test any other numbers because we can now solve the equation by dividing by (x − 1) and trying to solve the quadratic we get from the division.



Now we can factorise our expression as follows:

$$x^3 - 3x^2 - 2x + 4 = (x - 1)(x^2 - 2x - 4) = 0$$

It now remains for us to solve the quadratic equation.

$$x^2 - 2x - 4 = 0$$

We use the formula for quadratics with a = 1, b = −2 and c = −4.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$= \frac{2 \pm \sqrt{20}}{2}$$

$$= \frac{2 \pm \sqrt{4 \cdot 5}}{2}$$

$$= \frac{2 \pm 2\sqrt{5}}{2}$$

$$= \frac{\cancel{2}(1 \pm \sqrt{5})}{\cancel{2}}$$

$$= \underline{1 \pm \sqrt{5}}$$

We have now found all three solutions of the equation $x^3 - 3x^2 - 2x + 4 = 0$. They are:

$x = \underline{1}$

$x = \underline{1 + \sqrt{5}}$ $= 3.236$

$x = \underline{1 - \sqrt{5}}$ $= -1.236$

$= \underline{(x - 1)\ (x + 3.236)\ (\ x - 1.236)}$