Complex Numbers 'J' NOTATION

A Ten Minute Tutorial on using Complex Numbers

Introduction

Complex notation provides us with a simple yet powerful method of solving even the most complex of a.c. circuits. Complex notation allows us to represent electrical quantities that have both magnitude *and* direction (you will already know that in other contexts we call these vectors). The magnitude is simply the amount of resistance, reactance, voltage or current, etc. However, in order to specify the direction of the quantity, we use an operator to denote the phase shift relative to the reference quantity (this is usually current for a series circuit and voltage for a parallel circuit).

We call this operator j.

From studying complex numbers in mathematics, you will recall that every complex number consists of a real part and an imaginary part. In an electrical context, the real part is that part of the complex quantity that is in-phase with the reference quantity (current for most series circuits and voltage for most parallel circuits). The imaginary part, on the other hand, is that part of the complex quantity that is at 90 degrees to the reference.

The j-operator

You can think of the j-operator as a device that allows us to indicate a phase shift of 90 degrees. A phase shift of +90 deg. is represented by +j whilst a phase shift of -90deg. is represented by -j.

A phasor is simply an electrical vector. A vector, as you will doubtless recall, has magnitude (size) and direction (angle relative to some reference direction). The j-operator can be used to rotate a phasor. Each successive multiplication by j has the effect of rotating the phasor through a further 90 degrees.

The j-operator has a value which is equal to the square root of -1. Thus we can deduce $j^2 = j \times j = -1$, $j^3 = j \times j \times j = -j$, $j^4 = j \times j \times j \times j = +1$, and so on. Each time that we multiply a quantity by j we effectively rotate its phasor through 90 degrees.

Complex impedances

The j-operator provides us with a useful way of representing impedances. Any complex impedance can be represented by the relationship:

$Z = (R \pm j X)$

where Z represents impedance, R represents resistance and X represents reactance. All three quantities are, of course, measured in ohms.

The sign of the j term (either + or -) simply allows us to indicate whether the reactance is due to inductance (in which case the j term is positive, i.e. + j) or whether it is due to capacitance (in which case the j term is negative, i.e. - j).

Consider, for example, the following impedances:

1. $Z = 20 + j\,10$ this impedance comprises a resistance of 20 ohm connected in series with an inductive reactance (note the positive sign before the j-term) of 10 ohm.

2. $Z = 15 - j\,25$ this impedance comprises a resistance of 15 ohm connected in series with a capacitive reactance (note the positive sign before the j-term) of 25 ohm.

3. $Z = 30 + j\,0$ this impedance comprises a pure resistance of 30 ohm (there is no reactive component).

Voltages and currents can also take complex values. Consider the following:

1. $I = 2 + j\,0.5$ this current is the result of an in-phase component of 2A and a reactive component (at +90 deg.) of 0.5A.

2. $I = 1 - j\,1.5$ this current is the result of an in-phase component of 1A and a reactive component (at -90 deg.) of 1.5A.

3. $I = 3 + j\,0$ this current is in-phase and has a value of 3A.

Example 1

A current of 2A flows in an impedance of (100 + j 120) ohm. Derive an expression, in complex form, for the voltage that will appear across the impedance.

Since $V = I \times Z$

$V = 2 \times (100 + j\,120) = (200 + j\,240)$ V

Note that, in this example we have assumed that the supply current is the reference. In other words, it could be expressed in complex form as (2 + j 0) A.

Example 2

An impedance of (200 + j 100) ohm is connected to a 100V a.c. supply. Determine the current flowing and express your answer in complex form.

Since $I = V / Z$

$I = 100 / (200 + j\,100) = 100 \times (200 - j\,100) / (200 + j\,100) \times (200 - j\,100)$

(here we have multiplied the top and bottom by the complex conjugate)

$I = 100 \times (200 - j\,100) / (200^2 + 100^2)$

Thus I = (20,000 - j 10,000) / (40,000 + 10,000)

or I = (2 - j) / 5 = (0.4 - j 0.2) A

Note that, in this example we have assumed that the supply voltage is the reference. In other words, it could be expressed in complex form as (100 + j 0) V.

Problem

A current of (3 + j 4) A flows in a circuit when a voltage of (10 + j0) V is applied to it. Determine the impedance of the circuit and express your answer in complex form.

Possible answers are:

A) Z = 3.3 + j 2.5

B) Z = 1.2 - j 1.6

C) Z = 0.3 + j 0.4

Answer:

To find the impedance of the circuit you must divide the voltage by the current - both quantities being expressed in complex form.

Thus Z = V/I = (10 + j 0)/(3 + j 4)

To perform this division, you need to make use of the complex conjugate.

The complex conjugate of (3 + j 4) is (3 - j 4)

Thus Z = (10 + j 0)(3 - j 4) / (3 - j 4)(3 + j 4)

Now (10 + j 0) is the same as 10 + 0

Hence Z = 10(3 - j 4) / ($3^2 + 4^2$) = (30 - j 40) / 25 = 1.2 - j 1.6 —— THE ANSWER IS B.