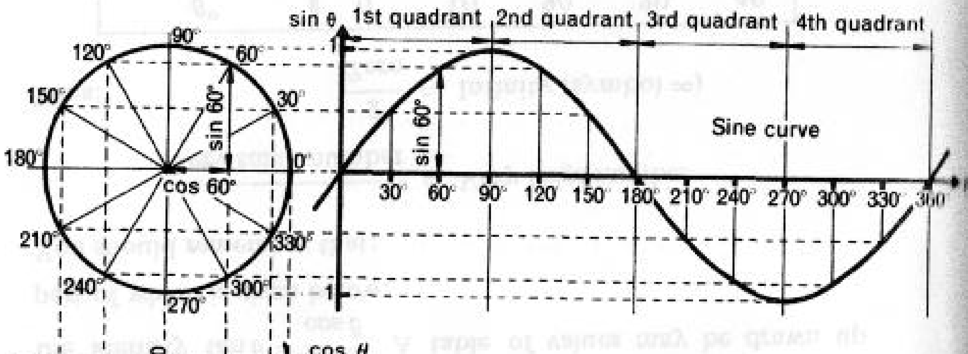
School of Engineering

SINE, COSINE AND TANGENT CURVES

A sine curve (or sine waveform) is the result of plotting vertically the values of $\sin\theta$ against θ horizontally (often called an angle base). The values of $\sin\theta$ may be found using a scientific calculator, but an alternative method is shown in Fig. 13.26. This makes use of the ideas expressed in Figs. 13.14 and 13.15.



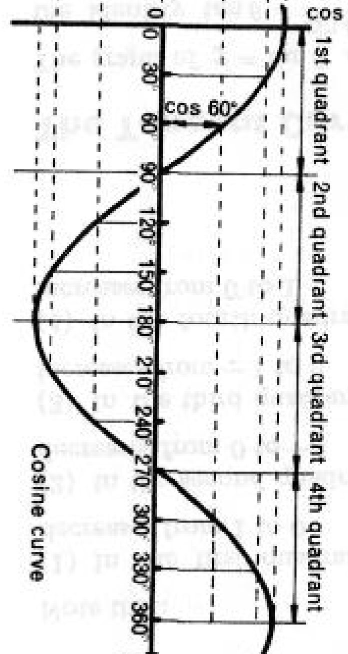


Fig. 13.26

The cosine curve may also be constructed as shown in Fig. 7.26. However it is usually drawn with the angle base horizontal (Fig. 13.28), in order that it may be compared with other trigonometrical curves.

The Sine Curve

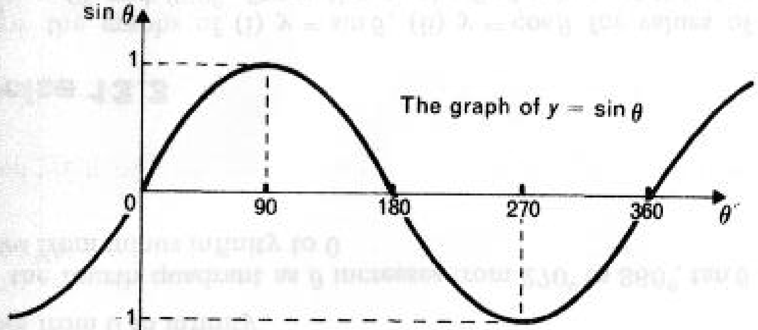


Fig. 13.27

The following features should be noted:

(1) In the first quadrant as θ increases from $0°$ to $90°$, $\sin\theta$ increases from 0 to 1

(2) In the second quadrant as θ increases from $90°$ to $180°$, $\sin\theta$ decreases from 1 to 0

(3) In the third quadrant as θ increases from $180°$ to $270°$, $\sin\theta$ decreases from 0 to -1

(4) In the fourth quadrant as θ increases from $270°$ to $360°$, $\sin\theta$ increases from -1 to 0

The Cosine Curve

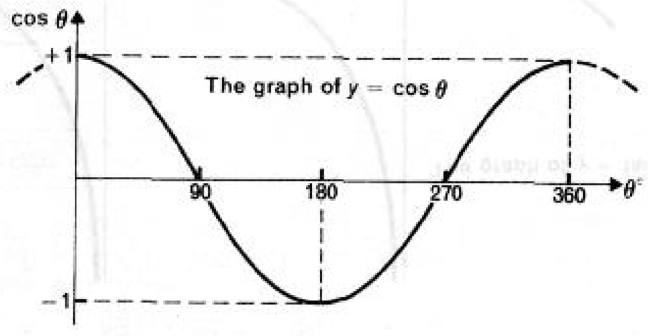


Fig. 13.28

Note that:

(1) In the first quadrant as θ increases from 0° to 90°, $\cos\theta$ decreases from 1 to 0

(2) In the second quadrant as θ increases from 90° to 180°, $\cos\theta$ decreases from 0 to -1

(3) In the third quadrant as θ increases from 180° to 270°, $\cos\theta$ increases from -1 to 0

(4) In the fourth quadrant as θ increases from 270° to 360°, $\cos\theta$ increases from 0 to 1

The Tangent Curve

The graph of $y = \tan\theta$ may be drawn using values obtained from the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. A table of values may be drawn up part of which is given below.

You should remember that:

$$\frac{1}{\text{Very small number}} = \text{Very large number}$$

Thus: $$\frac{1}{\text{Zero}} = \text{Infinity (symbol } \infty)$$

θ°	0	10	20	30	40
$\sin\theta$	0	0.174	0.342	0.500	0.643
$\cos\theta$	1	0.985	0.940	0.866	0.766
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	0	0.176	0.364	0.577	0.839

θ°	50	60	70	80	90
$\sin\theta$	0.766	0.866	0.940	0.985	1
$\cos\theta$	0.643	0.500	0.342	0.174	0
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	1.19	1.73	2.75	5.67	∞

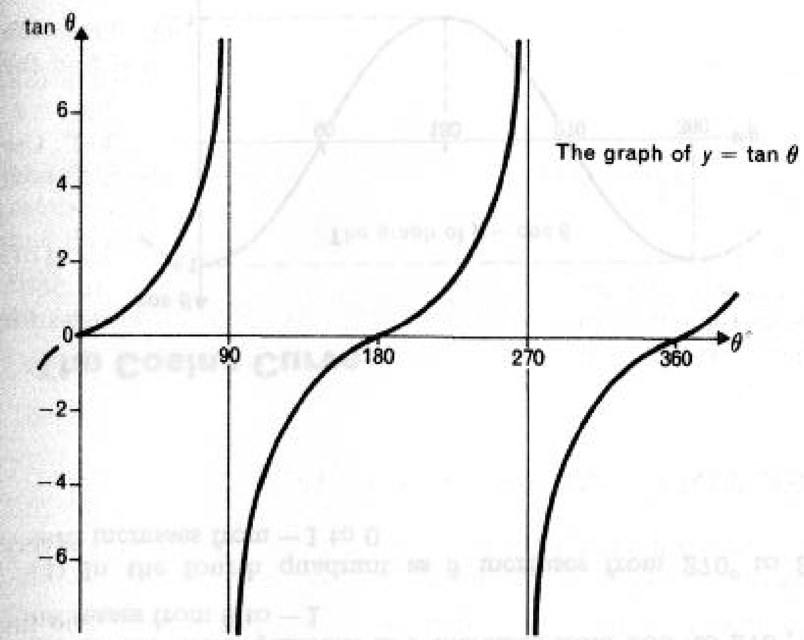


Fig. 13.29

Note that:

(1) In the first quadrant as θ increases from 0° to 90°, $\tan\theta$ increases from 0 to infinity

(2) In the second quadrant as θ increases from 90° to 180°, $\tan\theta$ increases from minus infinity to 0

(3) In the third quadrant as θ increases from 180° to 270°, $\tan\theta$ increases from 0 to infinity

(4) In the fourth quadrant as θ increases from 270° to 360°, $\tan\theta$ increases from minus infinity to 0

Exercise 13.3

1) Draw the graphs of (i) $y = \sin\theta$, (ii) $y = \cos\theta$ for values of θ between 0° and 360°. From the graphs find values of the sine and cosine of the angles:

(a) 38° (b) 72° (c) 142° (d) 108°
(e) 200° (f) [illegible]0° (g) 305° (h) 328°

Trig Idis

EX/ $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$= \frac{1-\cos 2\theta}{2}$$

if we use the trig identity.

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

in this instance. $A+B=\theta$

$$\cos(\theta+\theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$\cos 2\theta = \underbrace{\cos^2\theta} - \sin^2\theta$$

we also know that from other trig rule

that $\cos^2\theta = 1-\sin^2\theta$

$\therefore$ $\cos 2\theta = (1-\sin^2\theta) - \sin^2\theta$

expand $= 1 - 2\sin^2\theta$ where $\cos(2\theta) = 1-2\sin^2\theta$

Substitute $= \frac{1-(1-2\sin^2\theta)}{2}$

Power reduction formula $= \frac{\cancel{2}\sin^2\theta}{\cancel{2}}$

$= \sin^2\theta$

There are numerous applications of fundamental trigonometry to engineering situations, space has permitted us to look at just a few. You will be able to gain further practice by attempting the problems which will be found at the end of this section on trigonometry.

Trigonometric identities

Formula

(1) General identities

(i) $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$; $\sec\theta = \dfrac{1}{\cos\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$.

(ii) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\sin^2\theta + \cos^2\theta = 1$.

(iii) $\tan^2\theta + 1 = \sec^2\theta$; $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$.

(iv) $\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$.

(v) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$.

(vi) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$.

(2) Products to sums

(i) $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$.

(ii) $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$.

(iii) $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$.

(iv) $\sin A \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$.

(3) Sums to products

(i) $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

(ii) $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

(iii) $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

(iv) $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ where $A > B$.

(4) Doubles and squares

(i) $\sin 2A = 2\sin A\cos A$.

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.

(iii) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

(iv) $\cos^2 A + \sin^2 A = 1$.

(v) $\sec^2 A = 1$.

(vi) $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

(5) Hyperbolic doubles and squares

(i) $\sinh 2A = 2\sinh A\cosh A$.

(ii) $\cosh 2A = \cosh^2 A + \sinh^2 A = 2\cosh^2 A - 1 = 1 + 2\sinh^2 A$

(iii) $\tanh 2A = \dfrac{2\tanh A}{1 + \tanh^2 A}$.

(iv) $\cosh^2 A - \sinh^2 A = 1$.

(v) $\operatorname{sech}^2 A = 1 - \tanh^2 A$.

(vi) $\operatorname{cosech}^2 A = \coth^2 A - 1$.