WK11

COMPARISON OF LINEAR & ANGULAR RELATIONSHIPS

Linear Relationships	Angular Relationships
t = time (s)	t = time (s)
s = distance (m)	Θ = angle of rotation (rad)
u = initial linear velocity (m/s)	ω_i = initial angular velocity (rad/s)
v = final linear velocity (m/s)	ω_f = final angular velocity (rad/s)
a = linear acceleration (m/s^2)	α = angular acceleration (rad/s^2)
F = force (N)	T = torque (Nm)
m = mass (kg)	I = moment of inertia (kgm^2)
(For plain solid cylinder about polar axis $k = \frac{r}{\sqrt{2}}$)	($I = mk^2$ where k is the *radius of gyration* defined as the equivalent radius if all the mass was at one point rotating around the axis)
W = work (J)	W = work (J)
P = power (J/s or W)	P = power (J/s or W)
h = height above a datum (m)	
PE = potential energy (J)	PE = potential energy (J)
KE = kinetic energy (J)	KE = kinetic energy (J)

Transformation Equations

$s = r\Theta$ where r = radius (m)

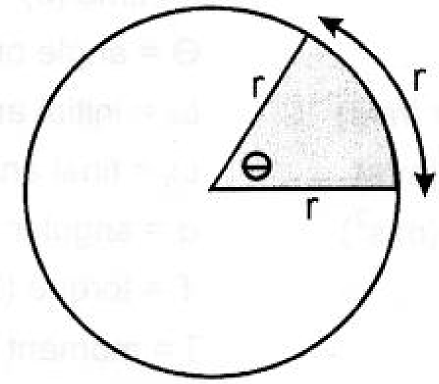
$v = r\,\omega_f,\; u = r\omega_i$

$a = r\alpha$

$v = u + at$	$\omega_f = \omega_i + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\Theta = \omega_i t + \frac{1}{2}\alpha t^2$
$s = \frac{1}{2}(u + v)t$	$\Theta = \frac{1}{2}(\omega_i + \omega_f)t$
$v^2 - u^2 = 2as$	$\omega_f^2 - \omega_i^2 = 2\alpha\Theta$
$F = ma$	$T = I\alpha$
$W = Fs$	$W = T\Theta$
$P = Fv$	$P = T\omega$
$PE = mgh$	
$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$

ANGULAR MOTION

An important concept in angular motion is the use of ***radian*** measure for angles.



Definition: A **RADIAN** is the angle (θ) subtended at the centre of a circle by an arc equal in length to the radius of the circle.

Relationship between Radians and Degrees

The circumference of a circle = $2\pi r$

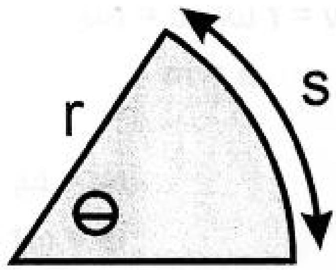
The number of radians in a circle is equal to the number of times r will divide into the circumference. That is

$$\text{Number of radians in a circle} = \frac{2\pi r}{r}$$

Therefore, there are 2π radians in a circle. Or 2π radians = 360°

One Radian is approximately 57°

Length of an Arc



If **θ** is measured in radians, then the length of the arc **s** is given by the formula

$$\mathbf{s = r \times \theta}$$

Angular Velocity

The Greek letter **ω** is used to represent angular velocity

So $$\omega = \frac{\text{Angle Turned Through}}{\text{Time Taken}}$$

$$\omega = \frac{\theta \text{ radians}}{t \text{ seconds}}$$

Rearranging this equation gives

$$\theta = \omega \times t$$

Revolutions per Minute (rpm)

Uniform angular velocity is often expressed in rpm. So if an object is rotating with uniform velocity of N rpm then

$$N \text{ rpm} = \frac{N}{60} \text{ revs/sec} = \frac{2\pi N}{60} \text{ rads/sec}$$

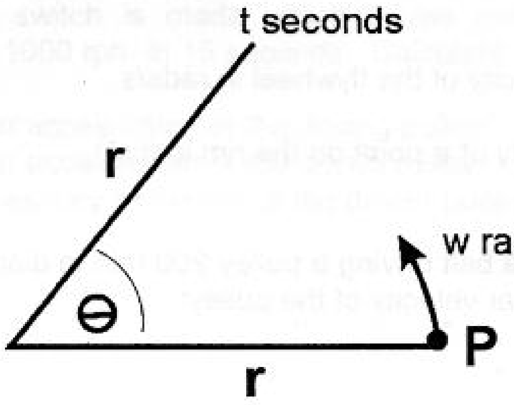
×60 ← 60 ← × ← 60 ← ×60

÷60 → ÷ → ÷60

Relationship between Angular and Linear Velocity

ie/60 rad/sec $\times \frac{60}{2\pi}$ = 572.9 rpm

50 rpm $\times \frac{2\pi}{60}$ = 5.24 rad/sec



A particle P rotates about a point a distance r at a constant speed of ω rads/sec

The distance travelled in time (t) seconds is given by:

$$s = r\theta$$

Also, since $\theta = \omega t$

Then $s = \omega t r$

The linear velocity of P is given by

$$v = s/t$$

so that $s = vt$

and $vt = \omega t r$

Hence $v = \omega r$

This means that

Linear Velocity = Angular Velocity x radius

PROBLEMS INVOLVING ANGULAR VELOCITY

1. A wheel rotates at 2800 rpm. What is the angular velocity in radians/sec?

2. A pulley is rotating with an angular velocity of 220 rads/sec. What is its speed in rpm?

3. The time taken for a wheel to make 490 revolutions was 35 seconds. Calculate the rotational speed of the wheel in radians per second.

4. An engine flywheel has a diameter of 0.4 metres and is rotating at 3000 rpm. Calculate:

    a. The angular velocity of the flywheel in rads/s

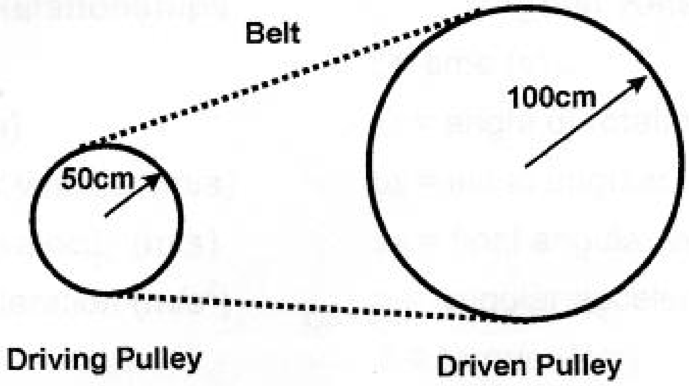
    b. The linear velocity of a point on the rim in ms^{-1}

5. The linear speed of a belt driving a pulley 200 mm in diameter is 5.5m/s. Determine the angular velocity of the pulley:

    a. In rads/sec

    b. In revs/min

Example



An electric motor is directly connected to a pulley of radius of 50 cm. A belt around the pulley is also passed around another pulley of 100 cm radius. Initially, the electric motor is stationary but when a switch is made which allows current to flow to the motor it accelerates from rest to 1000 rpm in 15 seconds. Calculate:

a. The angular acceleration of the driving pulley
b. The angular acceleration of the driven pulley
c. The linear velocity of the rim of the driven pulley.

ANGULAR VELOCITY PROBLEMS

① 2800 rpm → rad/s

$N_{rpm} \times \frac{2\pi}{60} = 2800 \times \frac{2\pi}{60} = \underline{293\,rad/s}$.

② $\omega = 220\,rad/s \rightarrow rpm$

$N_{rad/s} \times \frac{60}{2\pi} \rightarrow 220 \times \frac{60}{2\pi} = \underline{2100.84\,rpm}$

③ 490 rev, $t = 35\,sec$, $\omega = rad/s$?

$490 \times 2\pi = 3078.76$ radians

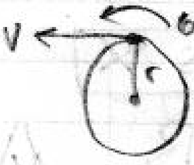
$\frac{3078.76}{35} = \underline{87.96\,rad/s} \times \frac{60}{2\pi} = \underline{840\,rpm}$

④ $\phi = 0.4m$ $N_{rpm} = 3000\,rpm$

$r = \frac{\phi}{2} = \frac{0.4}{2} = 0.2m$ $\therefore S = r\theta$

$= 0.2(2\pi)$

$N_{rpm} \times \frac{2\pi}{60} = 3000 \times \frac{2\pi}{60} = \underline{314.16\,rad/s}$ $= 1.256\,m \simeq$ 1 rotation distance



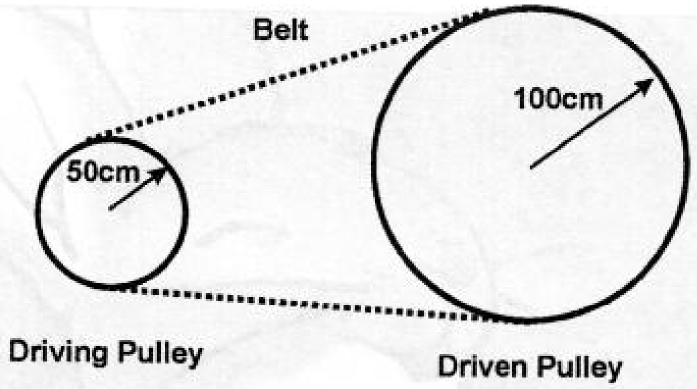
$\omega = \underline{314.16\,rad/s}$

$\therefore v = \omega r = 314.16(0.2) = \underline{62.83\,m/s}$

⑤ $\phi 200mm = 0.2m$ $v = 5.5m/s$ $v = \omega \cdot r \therefore$

$r = \frac{0.2}{2} = 0.1m$ $\therefore \omega = \frac{v}{r} = \frac{5.5}{0.1} = 55\,rad/s \times \frac{60}{2\pi} = \underline{525\,rpm}$

Example ANSWER



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a. The angular acceleration of the driving pulley
b. The angular acceleration of the driven pulley
c. The linear velocity of the rim of the driven pulley.

Conditions :- @ $t = 0 \rightarrow a = 0$

@ $t = 15 sec \rightarrow a = 1000 rpm$

pulley system $\frac{\text{driver}}{\text{driven}} = \frac{50}{100} = 0.5 = \frac{1}{2}$ (ratio)

a) $\omega_f = 1000 \times \frac{2\pi}{60} = 104.7 \text{ rad/s}$ angular velocity

$\omega_i = 0$ (at rest)

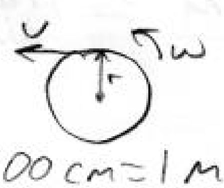
$\therefore \omega_f = \cancel{\omega_i}^{0} + \alpha t$

$\therefore \omega_f = \alpha \cdot t$

$\alpha = \frac{\omega_f}{t} = \frac{104.7}{15} = 6.98 \text{ rad/s}^2$ angular acceleration

b) $S_{driven} > S_{driver}$ $\therefore$ acceleration of driven gear, by the ratio 2:1 means that for 1 rotation of the driver, the driven pulley will have rotated $\frac{1}{2}$ a turn

Thus $\alpha_{driven} = \frac{1}{2} \alpha_{driver} = \frac{1}{2} 6.98 = \underline{3.49 \text{ rad/s}^2}$

c) 

$\omega_f = \cancel{\omega_i}^{0} + \alpha t$

$= 3.49(15)$

$= 52.35 \text{ rad/s}$

100 cm = 1 m

Linear velocity of the driven pulley:

$v = \omega r = 52.35(1) = 52.35 \text{ m/s} \times \frac{60}{2\pi} = 499.9 \simeq \underline{500 rpm}$