Hyperbolic functions.

hyperbolic sine → sinh → $\sinh x = \frac{e^{x} - e^{-x}}{2}$

hyperbolic cosine → cosh → $\cosh x = \frac{e^{x} + e^{-x}}{2}$

hyperbolic tangent → tanh → $\tanh x = \frac{\sinh}{\cosh}$

$= \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

hyperbolic cotangent → coth → $\coth x = \frac{1}{\tanh x}$

hyperbolic secant → sech → $\operatorname{sech} x = \frac{1}{\cosh x}$

hyperbolic cosecant → csch → $\operatorname{csch} x = \frac{1}{\sinh x}$

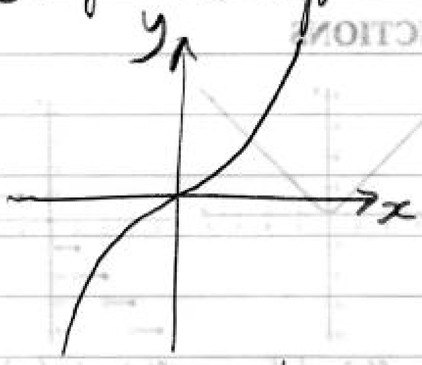
alternative form's;

$$\frac{1}{\tanh x} = \frac{\cosh x}{\sinh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

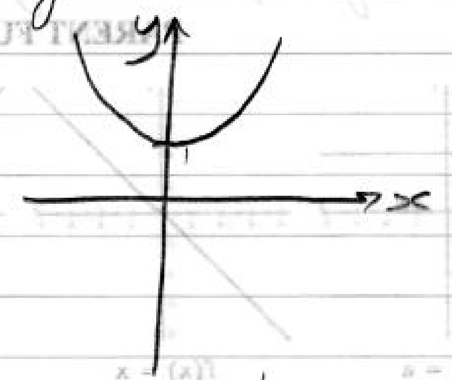
$$\frac{1}{\cosh x} = \frac{2}{e^{x} + e^{-x}}$$

$$\frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}}$$

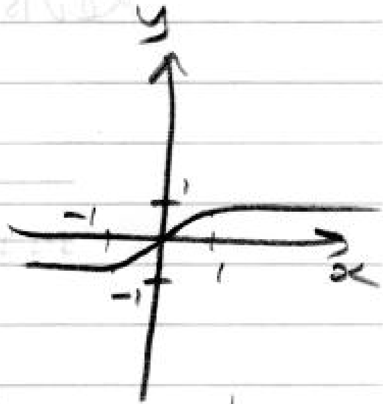
Graphed hyperbolic functions :—



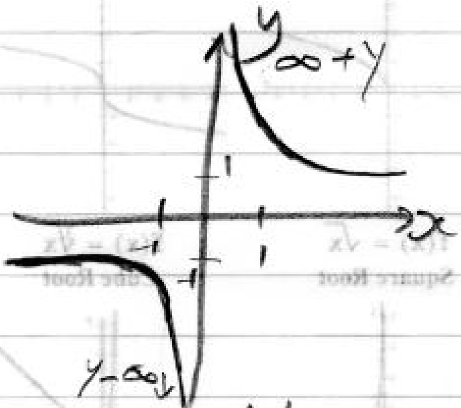
$y = \sinh x$



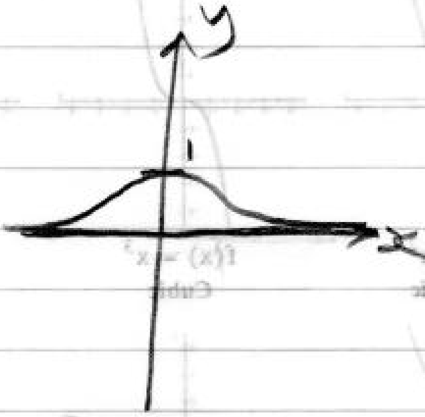
$y = \cosh x$



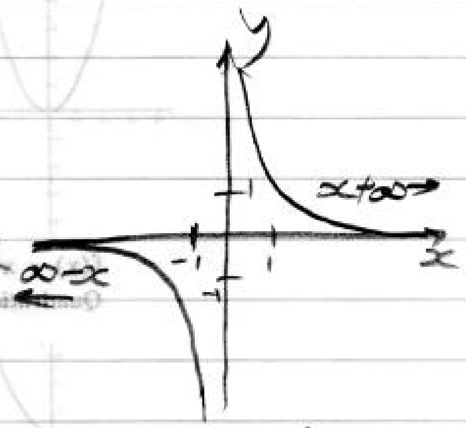
$y = \tanh x$



$y = \coth x$



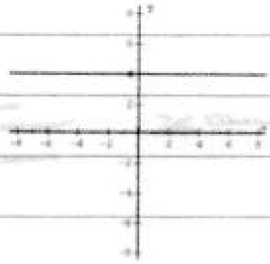
$y = \operatorname{sech} x$



$y = \operatorname{csch} x$

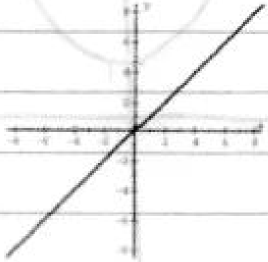
Revision of 'Parent functions'

PARENT FUNCTIONS



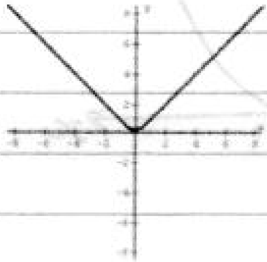
$f(x) = a$

Constant



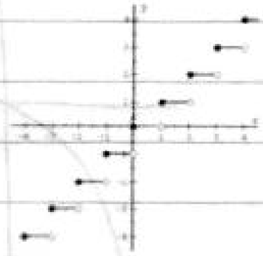
$f(x) = x$

Linear



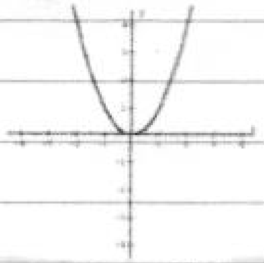
$f(x) = |x|$

Absolute Value



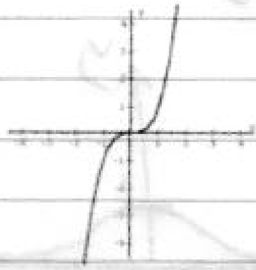
$f(x) = \text{int}(x) = [x]$

Greatest Integer



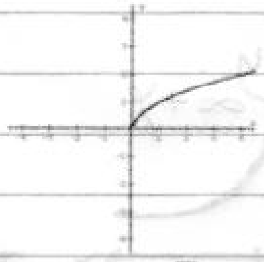
$f(x) = x^2$

Quadratic



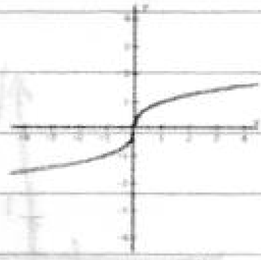
$f(x) = x^3$

Cubic



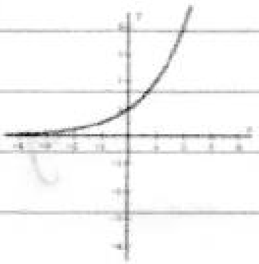
$f(x) = \sqrt{x}$

Square Root



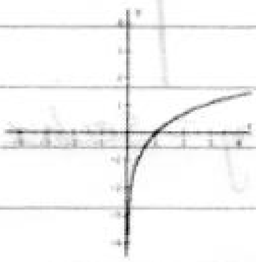
$f(x) = \sqrt[3]{x}$

Cube Root



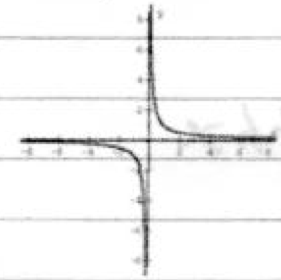
$f(x) = a^x$

Exponential



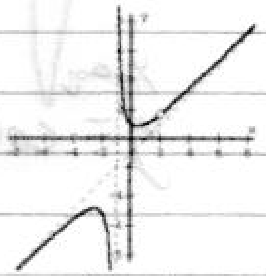
$f(x) = \log_a x$

Logarithmic



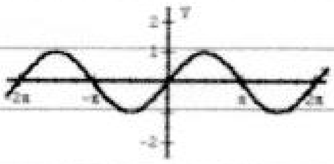
$f(x) = \dfrac{1}{x}$

Reciprocal

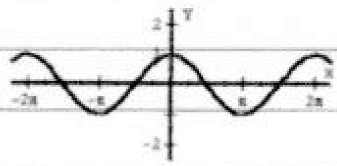


$f(x) = \dfrac{(x^2+1)(x-2)}{(x+1)(x-2)}$

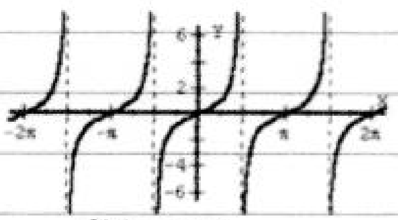
Rational



$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$

Trigonometric Functions

School of Engineering

Derivatives of Hyperbolic Functions

Derivative	Antiderivative/Integral
$\frac{d}{dx}(\cosh x) = \sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}(\sinh x) = \cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$	$\int \operatorname{sech}^2 x\,dx = \tanh x + C$
	$\int \tanh x\,dx = \ln(\cosh x) + C$
$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$	$\int \operatorname{csch}^2 x\,dx = -\coth x + C$
	$\int \coth x\,dx = \ln\lvert\sinh x\rvert + C$
$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$	$\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$
	$\int \operatorname{sech} x\,dx = \tan^{-1}\lvert\sinh x\rvert + C$
$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$	$\int \operatorname{csch} x \coth x\,dx = -\operatorname{csch} x + C$
	$\int \operatorname{csch} x\,dx = \ln\left\lvert\tanh\left(\frac{1}{2}x\right)\right\rvert + C$

Inverse derivatives of Hyperbolic Functions

Hyperbolic arcsine of $x = \sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) \quad -\infty < x < \infty$

Hyperbolic arccosine of $x = \cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) \quad x \geq 1$

Hyperbolic arctangent of $x = \tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad -1 < x < 1$

Hyperbolic arcsecant of $x = \operatorname{sech}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}-1}\right) \quad 0 < x \leq 1$

Hyperbolic arccosecant of $x = \operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}+1}\right) \quad x \neq 0$

Hyperbolic arccotangent of $x = \coth^{-1} x = \frac{1}{2}\ln\left(\frac{1-x}{1+x}\right) \quad x < -1 \text{ or } x > 1$

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Exercises

(5) Prove the following identities:

(a) $\cosh^2(x) - \sinh^2(x) = 1.$
(b) $\tanh^2(x) = 1 - \operatorname{sech}^2(x)$
(c) $\coth^2(x) = 1 + \operatorname{csch}^2(x)$
(d) $\sinh(a) + \sinh(b) = 2\sinh\left(\frac{a+b}{2}\right)\cosh\left(\frac{a-b}{2}\right).$

(a): Answer;

$$\begin{aligned}\cosh^2 x - \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2\\ &= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} = \frac{e^{2x}+2+e^{-2x}}{4} - \frac{e^{2x}-2+e^{-2x}}{4}\\ &= \frac{4}{4}\\ &= 1.\end{aligned}$$

Revision:

Definition

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$
$$\operatorname{csch} x = \frac{1}{\sinh x}$$
$$\operatorname{sech} x = \frac{1}{\cosh x}$$
$$\coth x = \frac{\cosh x}{\sinh x}$$

Hyperbolic Identities

$$\sinh(-x) = -\sinh x$$
$$\cosh(-x) = \cosh x$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\sinh(x+y) = \sinh x \cosh x + \cosh x \sinh y$$
$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

Derivatives of Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$
$$\frac{d}{dx}(\cosh x) = \sinh x$$
$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$
$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$
$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$
$$\frac{d}{dx}(\coth x) = -\operatorname{csch} x$$

Derivatives of Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$
$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, x > 1$$
$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, |x| < 1$$
$$\frac{d}{dx}(\operatorname{csch}^{-1} x) = \frac{-1}{|x|\sqrt{1-x^2}}, x \neq 0$$
$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}, 0 < x < 1$$
$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}, |x| > 1$$