STATICS

Simply supported beams

A simply supported beam is supported at two points in such a way that it is allowed to expand and bend freely. In practice the supports are often rollers. The loads on the beam may be concentrated at different points or uniformly distributed along the beam. Figure 1.30 shows concentrated loads only.

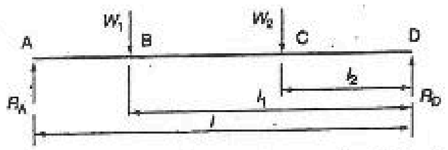


Figure 1.30 *Simply supported beam with concentrated loads*

The downward forces on a beam are said to be active loads, due to the force of gravity, whilst the loads carried by the supports are said to be reactive. When investigating the effects of loading, we often have to begin by calculating the supporting reactions. The beam is in static equilibrium under the action of these external forces, and so we proceed as follows:

1. Equate the sum of the turning moments, taken about the right hand support D, to zero. That is,

$$\Sigma M_D = 0$$
$$R_A l - W_1 l_1 - W_2 l_2 = 0$$

You can find R_A from this condition.

2. Equate vector sum of the vertical forces to zero. That is,

$$\Sigma F_V = 0$$
$$R_A + R_B - W_1 - W_2 = 0$$

You can find R_D from this condition.

Example 1.4

Calculate the support reactions of the simply supported beam shown in Figure 1.31.

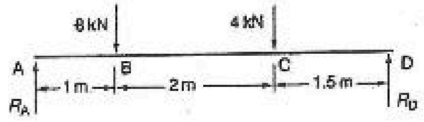


Figure 1.31

Take moments about the point D, remembering to use the sign convention that clockwise moments are positive and anticlockwise moments are negative. For equilibrium, $\Sigma M_D = 0$,

$$(R_A \times 4.5) - (8 \times 3.5) - (4 \times 1.5) = 0$$
$$4.5R_A - 28 - 6 = 0$$
$$R_A = \frac{28 + 6}{4.5} = 7.56 \text{ kN}$$

Equate the vector sum of the vertical forces to zero, remembering the sign convention that upward forces are positive and downward forces are negative. For equilibrium, $\Sigma F_V = 0$.

$$8 + 4 - 7.56 - R_D = 0$$
$$R_D = 8 + 4 - 7.56 = 4.44 \text{ kN}$$

Uniformly distributed loads (UDLs)

Uniformly distributed loads, or UDLs, are evenly spread out along a beam. They might be due to the beam's own weight, paving slabs or an asphalt surface. UDLs are generally expressed in kN per metre length, i.e. kN m^{-1}. This is also known as the 'loading rate'. Uniformly distributed loads are shown diagrammatically as in Figure 1.32.

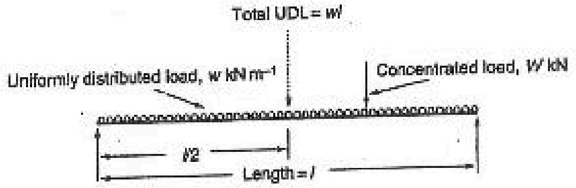


Figure 1.32 *Simply supported beam with concentrated and distributed loads*

The total UDL over a particular length l, of a beam is given by the product of the loading rate and the length. That is,

total UDL $= wl$

When you are equating moments to find the beam reactions, the total UDL is assumed to act at its centroid, i.e. at the centre of the length l. You can then treat it as just another concentrated load and calculate the support reactions in the same way as before.

Example 1.5

Calculate the support reactions of the simply supported beam shown in Figure 1.33.

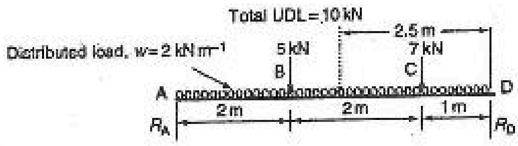


Figure 1.33

Begin by calculating the total UDL. Then, replace it by an equal concentrated load acting at its centroid. This is shown with a dotted line above.

Total UDL $= wl = 5 \times 2 = 10$ **kN**

You can now apply the conditions for static equilibrium. Begin by talking moments about the point D. For equilibrium, $\Sigma M_D = 0$,

$$(R_A \times 5) - (5 \times 3) - (10 \times 2.5) - (7 \times 1) = 0$$
$$5R_A - 15 - 25 - 7 = 0$$
$$R_A = \frac{15 + 25 + 7}{5} = 9.4 \text{ kN}$$

Now equate the vector sum of the vertical forces to zero. For equilibrium, $\Sigma F_V = 0$,

$$9.4 + R_D - 5 - 10 - 7 = 0$$
$$R_D = 5 + 10 + 7 - 9.4 = 12.6 \text{ kN}$$

Bending of beams

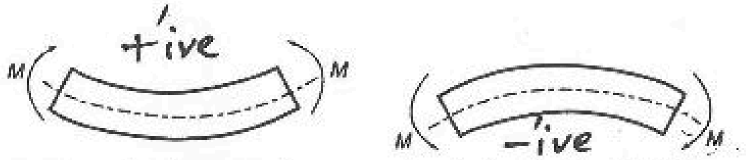
[redacted]The following sign convention is used:–

1. Clockwise moments to the left of a section are positive and anticlockwise moments are negative.
2. Anticlockwise moments to the right of a section are positive and clockwise moments are negative.

This gives rise to the idea of positive and negative bending as shown in Figure 1.36, and the variation of bending moment along a loaded beam can be plotted on a bending moment diagram.

Key point

Clockwise bending moments to the left of a section through a loaded beam are positive and anticlockwise moments are negative.



Positive bending—beam sagging   Negative bending—beam hogging

Figure 1.36 *Positive and negative bending*

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When you are plotting shear force and bending moment diagrams you will see that

1. The maximum bending moment always occurs where the shear force diagram changes sign.
2. The area of the shear force diagram up to a particular section gives the bending moment at that section.
3. Under certain circumstances, the bending moment changes sign and this is said to occur at a point of contraflexure.

At a point of contraflexure, where the bending moment is zero, the deflected shape of a beam changes from sagging to hogging or vice versa. The location of these points is of importance to structural engineers since it is here that welded, bolted or riveted joints can be made which will be free of bending stress. !!!

Key point

If a beam is to be made up of joined sections, it is good practice, when practicable, to position the joints at points of contraflexure where there are no bending stresses.

Example 1.6

Plot the shear force and bending moment distribution diagrams for the simple cantilever beam shown in Figure 1.37 and state the magnitude, nature and position of the maximum values of shear force, and bending moment.

shear force from A to B $= -5$ kN

shear force from B to C $= -5 - 3 = -8$ kN

maximum shear force $= -8$ kN between B and C

bending moment at A $= 0$

bending moment at B $= -(5 \times 2) = -10$ kN m

bending moment at C $= -(5 \times 3) - (3 \times 1) = -18$ kN m

maximum bending moment $= -18$ kN m at C

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As you can see, the shear force is negative over the whole length of the cantilever because there is a downward breaking force to the left of any section, i.e. negative shear.

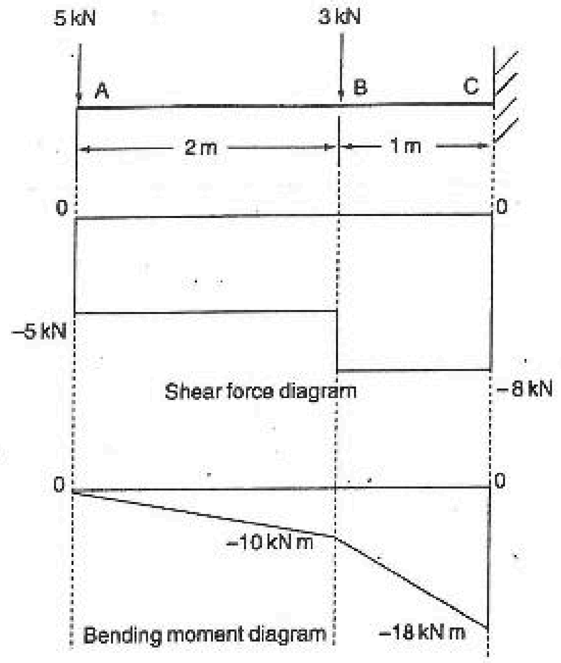


Figure 1.37

The bending moment is always zero at the free end of a simple cantilever and negative over the remainder of its length. You might think of a cantilever as being half of a hogging beam in which the bending moment at any section is anticlockwise, i.e. negative bending.

Example 1.7

Plot the shear force and bending moment distribution diagrams for the simply supported beam shown in Figure 1.38. State the magnitude, nature and position of the maximum values of shear force, and bending moment and the position of a point of contraflexure.

Begin by finding the support reactions:

For equilibrium, $\Sigma M_D = 0$,

$$(R_B \times 2) - (6 \times 3) - (10 \times 1) = 0$$

$$2\,R_B - 18 - 10 = 0$$

$$R_B = \frac{18 + 10}{2} = 14\ \text{kN}$$

Also, for equilibrium, $\Sigma F_V = 0$,

$$14 + R_D + 6 + 10 = 0$$

$$R_D = 6 + 10 - 14 = 2\ \text{kN}$$

EXAMPLE 1.7

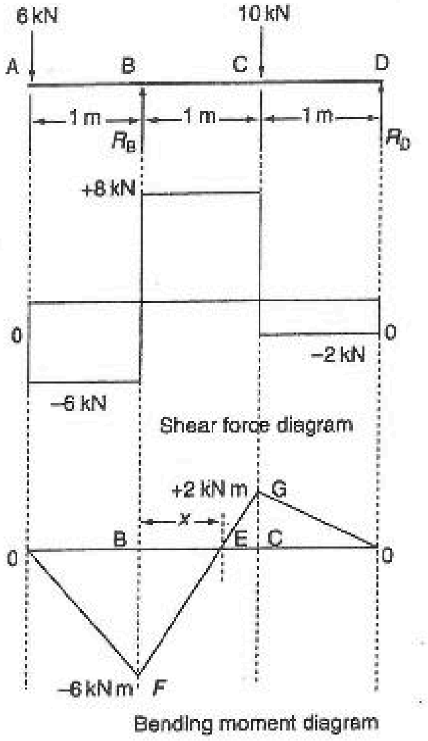


Figure 1.38

Next find the shear force values:

SF from A to B $= -6$ kN
SF from B to C $= -6 + 14 = +8$ kN
SF from C to D $= -6 + 14 - 10 = -2$ kN

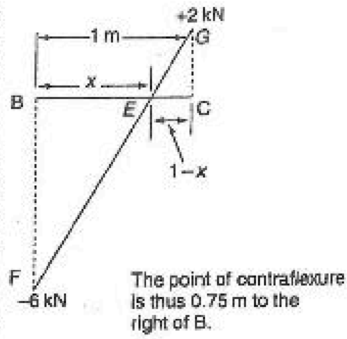
maximum shear force $= +8$ kN between B and C

Now find the bending moment values:

BM at A $= 0$
BM at B $= -(6 \times 1) = -6$ kN m
BM at C $= -(6 \times 2) + (14 \times 1) = +2$ kN m
BM at D $= 0$

maximum bending moment $= -6$ kN m at B

There is a point of contraflexure at E, where the bending moment is zero. To find its distance x, from B, consider the similar triangles BEF and ECG (Figure 1.39).



$$\frac{x}{6} = \frac{1-x}{2}$$
$$2x = 6(1 - x)$$
$$2x = 6 - 6x$$
$$8x = 6$$
$$x = \frac{6}{8} = \mathbf{0.75\ m}$$

Figure 1.39

If you examine the shear force and bending moment diagrams in the above examples, you will find that there is a relationship between shear force and bending moment. Calculate the area under the shear force diagram from the left hand end, up to any point along the beam. You will find that this is equal to the bending moment at that point. Try it, but remember to use the sign convention that areas above the zero line are positive and those below are negative.

Example 1.8

Plot the shear force and bending moment distribution diagrams for the cantilever shown in Figure 1.40 and state the magnitude and position of the maximum values of shear force and bending moment.

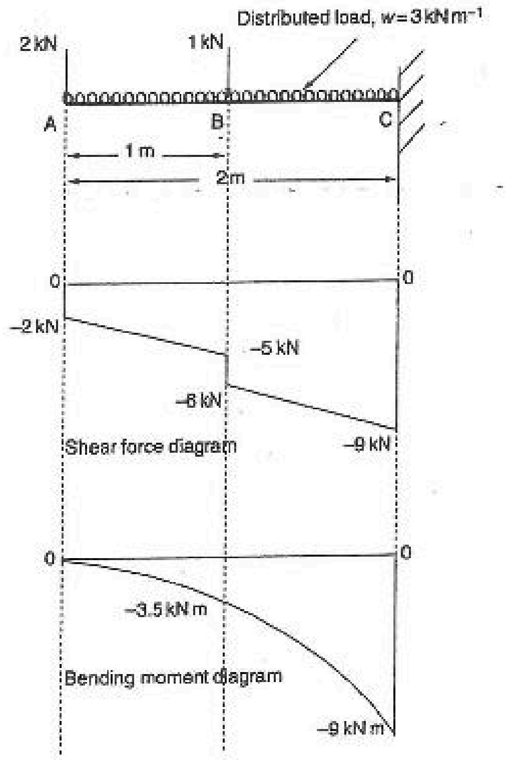


Figure 1.40

The presence of the UDL produces a gradually increasing shear force between the concentrated loads.

Finding shear force values:

SF immediately to right of A $= -2$ kN

SF immediately to left of B $= -2 - (3 \times 1) = -5$ kN

SF immediately to right of B $= -2 - 1 - (3 \times 1) = -6$ kN

SF immediately to left of C $= -2 - 1 - (3 \times 2) = -9$ kN

maximum SF $= -9$ kN immediately to left of C

The presence of the UDL produces a bending moment diagram with parabolic curves between the concentrated load positions.

Finding bending moment values:

BM at A $= 0$

BM at B $= -(2 \times 1) - (3 \times 1 \times 0.5) = -3.5$ kN m

BM at C $= -(2 \times 1) - (1 \times 1) - (3 \times 2 \times 1) = -9$ kN m

maximum bending moment $= -9$ kN m at C

Example 1.9

Plot the shear force and bending moment distribution diagrams for the simply supported beam shown in Figure 1.41. State the magnitude, nature and position of the maximum values of shear force and bending moment.

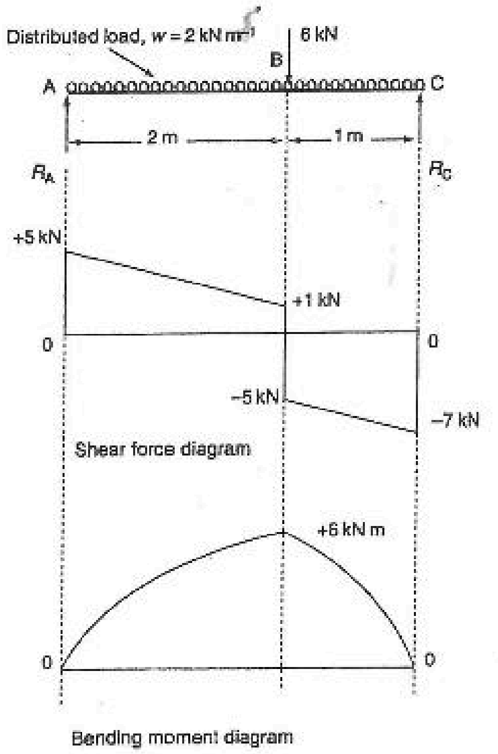


Figure 1.41

Finding support reactions:

For equilibrium, $\Sigma M_C = 0$,

$(R_A \times 3) - (6 \times 1) - (2 \times 3 \times 1.5) = 0$

$3R_A - 6 - 9 = 0$

$\boldsymbol{R_A = \frac{6+9}{3} = 5\,\text{kN}}$

Also for equilibrium, $\Sigma F_V = 0$,

$5 + R_C - 6 - (2 \times 3) = 0$

$5 + R_C - 6 - 6 = 0$

$\boldsymbol{R_C = 6 + 6 - 5 = 7\,\text{kN}}$

Finding the shear force values:

SF immediately to right of A $= +5$ kN

SF immediately to left of B $= 5 - (2 \times 2) = +1$ kN

SF immediately to right of B $= 5 - (2 \times 2) - 6 = -5$ kN

SF immediately to left of C $= 5 - (2 \times 3) - 6 = -7$ kN

maximum SF $= -7$ kN immediately to left of C

Finding bending moment values:

BM at A $= 0$

BM at B $= (5 \times 2) - (2 \times 2 \times 1) = +6$ kN m

BM at C $= 0$

maximum BM $= +6$ kN m at B

As you can see from the diagrams in Figure 1.41, the effect of the UDL is to produce a shear force diagram that slopes between the supports and the concentrated load. Its slope 2 kN m^{-1}, which is the uniformly distributed loading rate. The effect on the bending moment diagram is to produce parabolic curves between the supports and the concentrated load.

Test your knowledge 1.3

1. What is the difference between an active and a reactive load?
2. How do you define the shear force at any point on a loaded beam?
3. How do you define the bending moment at any point on a loaded beam?
4. What is a point of contraflexure?

Activity 1.3

The simply supported beam shown in Figure 1.42 is made in two sections which will be joined together at some suitable point between the supports. Draw the shear force and bending moment diagrams for the beam. State the maximum values of shear force and bending moment, and the positions where they occur. Where would be the most suitable point to join the two sections of the beam together?

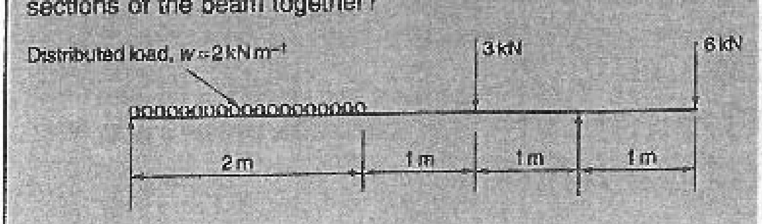


Figure 1.42

FURTHER BEAM

Problems [redacted]

Calculate resolution of forces & Moments, then Sketch the shear force and bending moment diagrams for the simply supported beams and cantilevers shown in Figures 1.43–1.47. Indicate the magnitude and nature of the maximum shear force and bending moment and the positions where they occur. Indicate also the position of any point of contraflexure.

1.

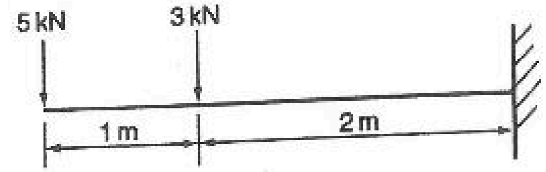


Figure 1.43

2.

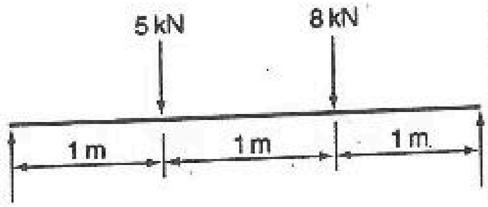


Figure 1.44

3.

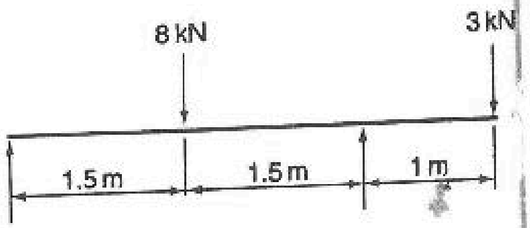


Figure 1.45

4.

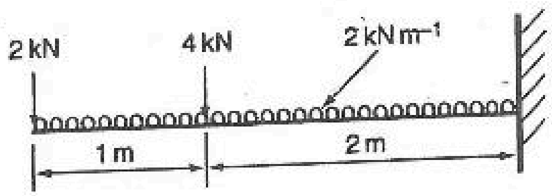


Figure 1.46

5.

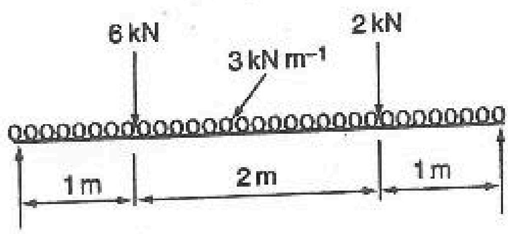


Figure 1.47